

Liquidity Black Holes*

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Abstract

Traders with short horizons and privately known trading limits interact in a market for a risky asset. Risk-averse, long horizon traders supply a downward sloping residual demand curve that face the short-horizon traders. When the price falls close to the trading limits of the short horizon traders, selling of the risky asset by any trader increases the incentives for others to sell. Sales become mutually reinforcing among the short term traders, and payoffs analogous to a bank run are generated. A “liquidity black hole” is the analogue of the run outcome in a bank run model. Short horizon traders sell because others sell. Using global game techniques, this paper solves for the unique trigger point at which the liquidity black hole comes into existence. Empirical implications include the sharp V-shaped pattern in prices around the time of the liquidity black hole.

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1. Introduction

Occasionally, financial markets experience episodes of turbulence of such an extreme kind that it appears to stop functioning. Such episodes are marked by a heavily one-sided order flow, rapid price changes, and financial distress on the part of many of the traders. The 1987 stock market crash is perhaps the most glaring example of such an episode, but there are other, more recent examples such as the collapse of the dollar against the yen on October 7th, 1998, and instances of distressed trading in some fixed income markets during the LTCM crisis in the summer of 1998. Practitioners dub such episodes as “liquidity holes” or, more dramatically, “liquidity black holes” (Taleb (1997, pp. 68-9), Persaud (2001)).

Liquidity black holes are not simply instances of large price changes. Public announcements of important macroeconomic statistics, such as the U.S. employment report or GDP growth estimates, are sometimes marked by large, discrete price changes at the time of announcement. However, such price changes are arguably the signs of a smoothly functioning market that is able to incorporate new information quickly. The market typically finds composure quite rapidly after such discrete price changes, as shown by Fleming and Remolona (1999) for the US Treasury securities market.

In contrast, liquidity black holes have the feature that they seem to gather momentum from the endogenous responses of the market participants themselves. Rather like a tropical storm, they appear to gather more energy as they develop. Part of the explanation for the endogenous feedback mechanism lies in the idea that the incentives facing traders undergo changes when prices change. For instance, market distress can feed on itself. When asset prices fall, some traders may get close to their trading limits and are induced to sell. But this selling pressure sets off further downward pressure on asset prices, which induces a further

round of selling, and so on. Portfolio insurance based on delta-hedging rules is perhaps the best-known example of such feedback, but similar forces will operate whenever traders face constraints on their behaviour that shorten their decision horizons. Daily trading limits and other controls on traders' discretion arise as a response to agency problems within a financial institution, and are there for good reason. However, they have the effect of shortening the decision horizons of the traders.

In what follows, we study traders with short decision horizons who have exogenously given trading limits. Their short decision horizon arises from the threat that a breach of the trading limit results in dismissal - a bad outcome for the trader. However, the trading limit of each trader is private information to that trader. Also, although the trading limits across traders can differ, they are closely correlated, *ex ante*. The traders interact in a market for a risky asset, where risk-averse, long horizon traders supply a downward sloping residual demand curve. When the price falls close to the trading limits of the short horizon traders, selling of the risky asset by any trader increases the incentives for others to sell. This is because sales tend to drive down the market-clearing price, and the probability of breaching one's own trading limit increases. This sharpens the incentives for other traders to sell. In this way, sales become reinforcing between the short term traders. In particular, the payoffs facing the short horizon traders are analogous to a bank run game. A "liquidity black hole" is the analogue of the run outcome in a bank run model. Short horizon traders sell because others sell.

If the trading limits were common knowledge, the payoffs have the potential to generate multiple equilibria. Traders sell if they believe others sell, but if they believe that others will hold their nerve and not sell, they will refrain from selling. Such multiplicity of equilibria is a well-known feature of the bank run model of Diamond and Dybvig (1983). However, when trading limits are not common

knowledge, as is more reasonable, the global game techniques of Morris and Shin (1998, 2003) and Goldstein and Pauzner (2000) can be employed to solve for the unique trigger point at which the liquidity black hole comes into existence.¹

The idea that the residual demand curve facing active traders is not infinitely elastic was suggested by Grossman and Miller (1988), who posited a role for risk-averse market makers who accommodate order flows and are compensated with higher expected return. Campbell, Grossman, and Wang (1993) find evidence consistent with this hypothesis by showing that returns accompanied by high volume tend to be reversed more strongly. Pastor and Stambaugh (2002) provide further evidence for this hypothesis by finding a role for a liquidity factor in an empirical asset pricing model, based on the idea that price reversals often follow liquidity shortages. Bernardo and Welch (2001) and Brunnermeier and Pedersen (2002) have used this device in modelling limited liquidity facing active traders².

More generally, the limited capacity of the market to absorb sales of assets has figured prominently in the literature on banking and financial crises (see Allen and Gale (2001), Gorton and Huang (2003) and Schnabel and Shin (2002)), where the price repercussions of asset sales have important adverse welfare consequences. Similarly, the inefficient liquidation of long assets in Diamond and Rajan (2000) has an analogous effect. The shortage of aggregate liquidity that such liquidations bring about can generate contagious failures in the banking system.

¹Global game techniques have been in use in economics for some time, but they are less well established in the finance literature. Some exceptions include Abreu and Brunnermeier (2003), Plantin (2003) and Bruche (2002).

²Lustig (2001) emphasizes solvency constraints in giving rise to a liquidity-risk factor in addition to aggregate consumption risk. Acharya and Pedersen (2002) develop a model in which each asset's return is net of a stochastic liquidity cost, and expected returns are related to return covariances with the aggregate liquidity cost (as well as to three other covariances). Gromb and Vayanos (2002) build on the intuitions of Shleifer and Vishny (1997) and show that margin constraints have a similar effect in limiting the ability of arbitrageurs to exploit price differences. Holmström and Tirole (2001) propose a role for a related notion of liquidity arising from the limited pledgeability of assets held by firms due to agency problems.

Some market microstructure studies show evidence consistent with an endogenous trading response that magnifies the initial price change. Cohen and Shin (2001) show that the US Treasury securities market exhibit evidence of positive feedback trading during periods of rapid price changes and heavy order flow. Indeed, even for macroeconomic announcements, Evans and Lyons (2003) find that the foreign exchange market relies on the order flow of the traders in order to interpret the significance of the macro announcement. Hasbrouck (2000) finds that a flow of new market orders for a stock are accompanied by the withdrawal of limit orders on the opposite side. Danielsson and Payne's (2001) study of foreign exchange trading on the Reuters 2000 trading system shows how the demand or supply curve disappears from the market when the price is moving against it, only to reappear when the market has regained composure. The interpretation that emerges from these studies is that smaller versions of such liquidity gaps are pervasive in active markets - that the market undergoes many "mini liquidity gaps" several times per day.

The next section presents the model. We then proceed to solve for the equilibrium in the trading game using global game techniques. We conclude with a discussion of the empirical implications and the endogenous nature of market risk.

2. Model

An asset is traded at two consecutive dates, and then is liquidated. We index the two trading dates by 1 and 2. The liquidation value of the asset at date 2 when viewed from date 0 is given by

$$v + z \tag{2.1}$$

where v and z are two independent random variables. z is normally distributed with mean zero and variance σ^2 , and is realized after trading at date 2. v is

realized after trading at date 1. We do not need to impose any assumptions on the distribution of v . The important feature for our exercise is that, at date 1 (after the realization of v), the liquidation value of the asset is normal with mean v and variance σ^2 .

There are two groups of traders in the market, and the realization of v at date 1 is common knowledge among all of them. There is, first, a continuum of risk neutral traders of measure 1. Each trader holds 1 unit of the asset. We may think of them as proprietary traders (e.g. at an investment bank or hedge fund). They are subject to an incentive contract in which their payoff is proportional to the final liquidation value of the asset. However, these traders are also subject to a loss limit at date 1, as will be described in more detail below. If a trader's loss between dates 0 and 1 exceeds this limit, then the trader is dismissed. Dismissal is a bad outcome for the trader, and the trader's decision reflects the tradeoff between keeping his trading position open (and reaping the rewards if the liquidation value of the asset is high), against the risk of dismissal at date 1 if his loss limit is breached at date 1. We do not model explicitly the agency problems that motivate the loss limit. The loss limit is taken to be exogenous for our purpose.

Alongside this group of risk-neutral traders is a risk-averse market-making sector of the economy. The market-making sector provides the residual demand curve facing the risk-neutral traders as a whole, in the manner envisaged by Grossman and Miller (1988) and Campbell, Grossman and Wang (1993).

We represent the market-making sector by means of a representative trader with constant absolute risk aversion γ who posts limit buy orders for the asset at date 1 that coincides with his competitive demand curve. At date 1 (after v is realized), the liquidation value of the asset is normally distributed with mean v and variance σ^2 . From the linearity of demand with Gaussian uncertainty

and exponential utility, the market-making sector's limit orders define the linear residual demand curve:

$$d = \frac{v - p}{\gamma\sigma^2}$$

where p is the price of the asset at date 1. Thus, if the aggregate net supply of the asset from the risk-neutral traders is s , price at date 1 satisfies

$$p = v - cs \tag{2.2}$$

where c is the constant $\gamma\sigma^2$. Since the market-making sector is risk-averse, it must be compensated for taking over the risky asset at date 1, so that the price of the asset falls short of its expected payoff by the amount cs .

2.1. Loss limits

In the absence of any artificial impediments, the efficient allocation is for the risk-neutral traders to hold all of the risky asset. However, the risk-neutral traders are subject to a loss limit that constrains their actions. The loss limit is a trigger price or “stop price” q_i for trader i such that if

$$p < q_i$$

then trader i is dismissed at date 1. Dismissal is a bad outcome for the trader, and results in a payoff of 0. The loss limits of the traders should be construed as being determined in part by the overall risk position and portfolio composition of their employers. Loss limits therefore differ across traders, and information regarding such limits are closely guarded. Among other things, the loss limits fail to be common knowledge among the traders. This will be the crucial feature of our model that drives the main results. We will also assume that, conditional on being dismissed, the trader prefers to maximize the value of his trading book. The idea here is that the trader is traded more leniently if the loss is smaller.

We will model the loss limits as random variables that are closely correlated across the traders. Trader i 's loss limit q_i is given by

$$q_i = \theta + \eta_i \tag{2.3}$$

where θ is a uniformly distributed random variable with support $[\underline{\theta}, \bar{\theta}]$, representing the common component of all loss limits. The idiosyncratic component of i 's loss limit is given by the random variable η_i , which is uniformly distributed with support $[-\varepsilon, \varepsilon]$, and where η_i and η_j for $i \neq j$ are independent, and η_i is independent of θ . Crucially, trader i knows only of his own loss limit q_i . He must infer the loss limits of the other traders, based on his knowledge of the joint distribution of $\{q_j\}$, and his own loss limit q_i .

2.2. Execution of sell orders

The trading at date 1 takes place by matching the sales of the risk-neutral traders with the limit buy orders posted by the market-making sector. However, the sequence in which the sell orders are executed is not under the control of the sellers. We will assume that if the aggregate sale of the asset by the risk-neutral traders is s , then a seller's place in the queue for execution is uniformly distributed in the interval $[0, s]$. Thus the expected price at which trader i 's sell order is executed is given by

$$v - \frac{1}{2}cs \tag{2.4}$$

and depends on the aggregate sale s . This feature of our model captures two ingredients. The first is the idea that the price received by a seller depends on the amount sold by other traders. When there is a flood of sell orders (large s), then the sale price that can be expected is low. The second ingredient is the departure from the assumption that the transaction price is known with certainty when a trader decides to sell. Even though traders may have a good indication

of the price that they can expect by selling (say, through indicative prices), the actual execution price cannot be guaranteed, and will depend on the overall selling pressure in the market. This second feature - the uncertainty of transactions price - is an important feature of a market under stress, and is emphasized by many practitioners (see for instance, Kaufman (2000, pp.79-80), Taleb (1997, 68-9)).

The payoff to a seller now depends on whether the execution price is high enough as not to breach the loss limit. Let us denote by \hat{s}_i the largest value of aggregate sales s that guarantees that trader i can execute his sell order without breaching the loss limit. That is, \hat{s}_i is defined in terms of the equation:

$$q_i = v - c\hat{s}_i \tag{2.5}$$

where the expression on the right hand side is the lowest possible price received by a seller when the aggregate sale is \hat{s}_i . Thus, whenever $s \leq \hat{s}_i$, trader i 's expected payoff to selling is given by (2.4). However, when $s > \hat{s}_i$, there is a positive probability that the loss limit is breached, which leads to the bad payoff of 0. When $s > \hat{s}_i$, trader i 's expected payoff to selling is

$$\frac{\hat{s}_i}{s} \left(v - \frac{1}{2}c\hat{s}_i \right) \tag{2.6}$$

If trader i decides to hold on to the asset, then the payoff is given by the liquidation value of the asset at date 2 if the loss limit is not breached, and 0 if it is breached. Thus, the expected payoff to trader i of holding the asset, as a function of aggregate sales s , is

$$u(s) = \begin{cases} v & \text{if } s \leq \hat{s}_i \\ 0 & \text{if } s > \hat{s}_i \end{cases} \tag{2.7}$$

Bringing together (2.4) and (2.6), we can write the expected payoff of trader i

from selling the asset as

$$w(s) = \begin{cases} v - \frac{1}{2}cs & \text{if } s \leq \hat{s}_i \\ \frac{\hat{s}_i}{s} (v - \frac{1}{2}cs) & \text{if } s > \hat{s}_i \end{cases} \quad (2.8)$$

The payoffs are depicted in Figure 2.1. Holding the asset does better when $s < \hat{s}_i$, but selling the asset does better when $s > \hat{s}_i$. The trader's optimal action depends on the density over s . We now solve for equilibrium in this trading game.

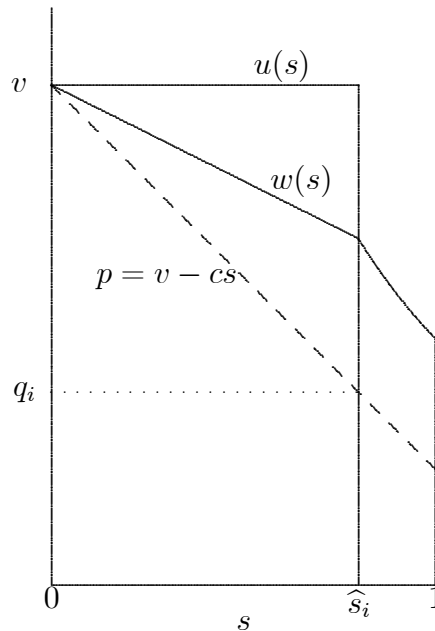


Figure 2.1: Payoffs

3. Equilibrium

At date 1, v is realized, and is common knowledge among all traders. Thus, at date 1, it is common knowledge that the liquidation value at date 2 has mean v

and variance σ^2 . Each trader decides whether to sell or hold the asset on the basis of the realization of v and his own loss limit. Trader i 's strategy is a function

$$(v, q_i) \mapsto \{\text{hold, sell}\}$$

that maps realizations of v and q_i to a trading decision. When v is either very high or very low, trader i has a dominant action. When v is very high relative to q_i (so that the realization of v is considerably higher than the loss limit for i), trader i will prefer to hold. In particular, since the period 1 price p cannot fall below $v - c$, trader i 's dominant action is to hold when:

$$v \geq q_i + c \tag{3.1}$$

This is because the loss limit for trader i will not be breached even if all other traders sell. Conversely, when v is so low that

$$v < q_i \tag{3.2}$$

then the loss limit is breached even if all other traders hold. Given our assumption that the traders prefers to maximize the value of his trading book conditional on being dismissed, selling is the dominant action when $v < q_i$. However, for intermediate values of v where

$$q_i \leq v < q_i + c \tag{3.3}$$

trader i 's optimal action depends on the incidence of selling by other traders. If trader i believes that others are selling, he will sell also. If, however, the others are not selling, then he will hold. If the loss limits were common knowledge, then such interdependence of actions would lead to multiple equilibria, and an indeterminacy in the predicted outcome. When the loss limits are not common knowledge (as in our case), we can largely eliminate the multiplicity of equilibria through global game techniques.

In particular, we will solve for the unique equilibrium in threshold strategies in which trader i has the threshold $v^*(q_i)$ for v that depends on his own loss limit q_i such that the equilibrium strategy is given by

$$(v, q_i) \mapsto \begin{cases} \text{hold} & \text{if } v \geq v^*(q_i) \\ \text{sell} & \text{if } v < v^*(q_i) \end{cases} \quad (3.4)$$

In other words, $v^*(q_i)$ is the trigger level of v for trader i such that he sells if and only if v falls below this critical level. We will show that there is precisely one equilibrium of this kind, and proceed to solve for it by solving for the trigger points $\{v^*(q_i)\}$. Our claim can be summarized in terms of the following theorem.

Theorem 1. *There is an equilibrium in threshold strategies where the threshold $v^*(q_i)$ for trader i is given by the unique value of v that solves*

$$v - q_i = c \exp \left\{ \frac{q_i - v}{2(v + q_i)} \right\} \quad (3.5)$$

There is no other threshold equilibrium.

The left hand side of (3.5) is increasing in v and passes through the origin, while the right hand side is decreasing in v and passes through $(0, c)$, so that there is a unique solution to (3.5). At this solution, we must have $v - q_i > 0$, so that the trigger point $v^*(q_i)$ is strictly above the loss limit q_i . Traders adopt a pre-emptive selling strategy in which the trigger level leaves a “margin for prudence”. The intuition here is that a trader anticipates the negative consequences of *other* traders selling. Other traders’ pre-emptive selling strategy must be met by a pre-emptive selling strategy on my part. In equilibrium, every trader adopts an aggressive, pre-emptive selling strategy because others do so. If the traders have long decision horizons, they can ignore the short-term fluctuations in price and hold the asset for its fundamental value. However, traders subject to a loss limit have a short decision horizon. Even though the fundamentals are good, short term

price fluctuations can cost him his job. Thus, loss limits inevitably shorten the decision horizon of the traders. The fact there there is a pre-emptive equilibrium of this kind is perhaps not so remarkable. However, what is of interest is the fact that there is no other threshold equilibrium. In particular, the “nice” strategy in which the traders disarm by collectively lowering their threshold points $v^*(q_i)$ down to their loss limits q_i cannot figure in any equilibrium behaviour.

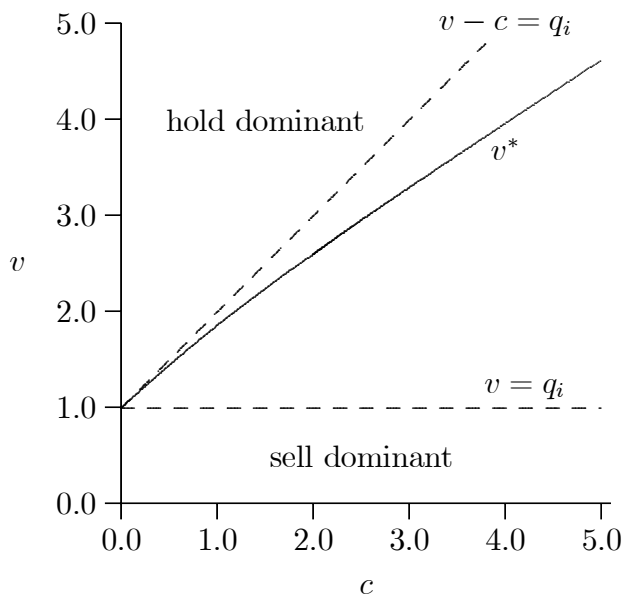


Figure 3.1: v^* as a function of c . $q_i = 1$

Figure 3.1 plots v^* as a function of the parameter c as given by (3.5), while fixing $q_i = 1$. Recall that $c = \gamma\sigma^2$, where γ is the coefficient of absolute risk aversion. We can see that the critical value v^* can be substantially higher than the loss limit (given by 1). When v is very high, so that $v - c > q_i$, holding the asset is the dominant action. This dominance region is the area above the upward sloping dashed line in figure 3.1. Conversely when $v < q_i$, the dominant action is to sell, and this area is indicated as the region below the horizontal dashed line.

The large “wedge” between these two dominance regions is the region in which the outcome depends on the resolution of the strategic trading game between the traders. The equilibrium trigger point v^* bisects this wedge, and determines whether trader i holds or sells. The solid line plots the equilibrium trigger point given by the solution to (3.5).

Technically, the global game analysed here does not conform to the canonical case discussed in Morris and Shin (2003) in which the payoffs satisfy strategic complementarity, and uniqueness can be proved by the iterated deletion of dominated strategies. In our game, the payoff difference between holding and selling is not a monotonic function of s . We can see this best from figure 2.1. The payoff difference rises initially, but then drops discontinuously, and then rises thereafter, much like the bank run game of Goldstein and Pauzner (2000). Our argument for the uniqueness of the threshold equilibrium rests on the interaction between strategic uncertainty (uncertainty concerning the actions of other traders) and fundamental uncertainty (uncertainty concerning the fundamentals). Irrespective of the severity of fundamental uncertainty, the strategic uncertainty persists in equilibrium, and the pre-emptive action of the traders reflects the optimal response to strategic uncertainty. Our solution method below will bring this feature out explicitly.

3.1. Strategic uncertainty

The payoff difference between holding the asset and selling the asset when aggregate sales are s is given by $u(s) - w(s)$. The expected payoff advantage of holding the asset over selling it is given by

$$\int_0^1 f(s|v, q_i) [u(s) - w(s)] ds$$

where $f(s|v, q_i)$ is the density over the equilibrium value of s (the proportion of traders who sell) conditional on v and trader i 's own loss limit q_i . Trader i will

hold if the integral is positive, and sell if it is negative. Thus, a direct way to solve for our equilibrium is to solve for the density $f(s|v, q_i)$.

It is convenient to view the trader's threshold strategy as the choice of a threshold for q_i as a function of v . Thus, let us fix v and suppose that all traders follow the threshold strategy around q^* , so that trader i sells if $q_i > q^*$ and holds if $q_i \leq q^*$. Suppose that trader i 's loss limit q_i happens to be exactly q^* . We will derive trader i 's subjective density over the aggregate sales s . Since the traders have unit measure, aggregate sales s is given by the proportion of traders who sell. From trader i 's point of view, s is a random variable with support on the unit interval $[0, 1]$. The cumulative distribution function over s viewed from trader i 's viewpoint can be obtained from the answer to the following question.

“My loss limit is q^* . What is the probability that s is less than z ?” (Q)

The answer to this question will yield $F(z|q^*)$ - the probability that the proportion of traders who sell is at most z , conditional on $q_i = q^*$. Since all traders are hypothesized to be using the threshold strategy around q^* , the proportion of traders who sell is given by the proportion of traders whose loss limits have realizations to the right of q^* . When the common element of the loss limits is θ , the individual loss limits are distributed uniformly over the interval $[\theta - \varepsilon, \theta + \varepsilon]$. The traders who sell are those whose loss limits are above q^* . Hence,

$$s = \frac{\theta + \varepsilon - q^*}{2\varepsilon}$$

When do we have $s < z$? This happens when θ is low enough, so that the area under the density to the right of q^* is squeezed to a size below z . There is a value of θ at which s is precisely equal to z . This is when $\theta = \theta^*$, where

$$\theta^* = q^* - \varepsilon + 2\varepsilon z$$

We have $s < z$ if and only if $\theta < \theta^*$. Thus, we can answer question (Q) by finding the posterior probability that $\theta < \theta^*$.

For this, we must turn to trader i 's posterior density over θ conditional on his loss limit being q^* . This posterior density is uniform over the interval $[q^* - \varepsilon, q^* + \varepsilon]$. This is because the ex ante distribution over θ is uniform and the idiosyncratic element of the loss limit is uniformly distributed around θ . The probability that $\theta < \theta^*$ is then the area under the posterior density over θ to the left of θ^* . This is,

$$\begin{aligned} & \frac{\theta^* - (q^* - \varepsilon)}{2\varepsilon} & (3.6) \\ = & \frac{q^* - \varepsilon + 2\varepsilon z - (q^* - \varepsilon)}{2\varepsilon} \\ = & z \end{aligned}$$

In other words, the probability that $s < z$ conditional on loss limit q^* is exactly z . The cumulative distribution function $F(z|q^*)$ is the identity function:

$$F(z|q^*) = z \quad (3.7)$$

The density over s is then obtained by differentiation.

$$f(s|q^*) = 1 \quad \text{for all } s \quad (3.8)$$

The density over s is uniform. The noteworthy feature of this result is that the constant ε does not enter into the expression for the density over s . No matter how small or large is the dispersion of loss limits, s has the uniform density over the unit interval $[0, 1]$. In the limit as $\varepsilon \rightarrow 0$, every trader's loss limit converges to θ . Thus, fundamental uncertainty disappears. Everyone's loss limit converges to the common element θ , and everyone knows this fact. And yet, even as fundamental uncertainty disappears, the strategic uncertainty is unchanged.

3.2. Solving for Equilibrium Threshold

Having found the conditional density over s at the threshold point q^* , we can now return to the payoffs of the game. We noted earlier that the expected payoff advantage to holding the asset is given by

$$\int_0^1 f(s|v, q_i) [u(s) - w(s)] ds$$

At the threshold point q^* , we have just shown that the density $f(s|v, q_i)$ is uniform. In addition, the trader is indifferent between holding and selling. Thus, at the threshold point, we have

$$\int_0^1 [u(s) - w(s)] ds = 0$$

From this equation, we can solve for the threshold point. Figure 3.2 illustrates the argument. The integral of the payoff difference with respect to a uniform density over s must be equal to zero. This means that the area labelled A in figure 3.2 must be equal to the area labelled B .

Substituting in the expressions for (2.7) and (2.8), and noting that $\hat{s} = (v - q^*)/c$, we have

$$\frac{1}{2}c \int_0^{\frac{v-q^*}{c}} s ds = \frac{(v-q^*)(v+q^*)}{2cs} \int_{\frac{v-q^*}{c}}^1 \frac{1}{s} ds$$

which simplifies to

$$v - q^* = 2(v + q^*) \log \frac{c}{v - q^*}$$

Re-arranging this equation gives (3.5) of theorem 1. There is a unique solution to this equation as already noted, where $v > q^*$.

So far, we have shown that if all traders follow the threshold strategy around q^* , then a trader is indifferent between holding and selling given the threshold loss limit q^* . We must show that if $q_i > q^*$, trader i prefers to sell, and if $q_i < q^*$, trader i prefers to hold. This step of the argument is presented in the appendix.

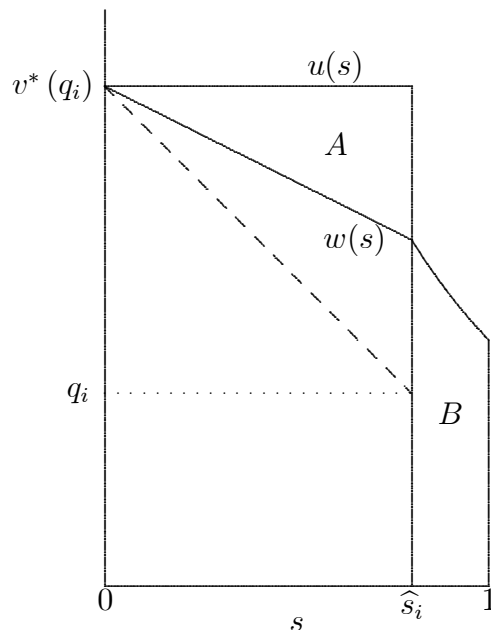


Figure 3.2: Expected Payoffs in Equilibrium

3.3. Normal distribution over types

Although our argument has been given in the context of the uniform distribution over loss limits, this particular distributional assumption turns out not to be important. What matters for the argument is the shape of the strategic uncertainty facing the traders, in particular, the density over the proportion who sell. Provided that the idiosyncratic terms are small, the assumption of uniformity is without loss of generality (see Morris and Shin (2003, section 2)). We illustrate the point by showing how we would obtain the same solution (3.5) for the threshold point when the density of loss limits is Gaussian.

Thus, suppose that θ is distributed normally with mean $\bar{\theta}$ and precision α , and q_i is the sum $\theta + \varepsilon_i$ where ε_i is normal with mean zero and precision β , where $\{\varepsilon_i\}$ are independent across i , and independent of θ . We can then pose question (Q)

in this new context. The answer is given by the area under the normal density to the right of the threshold point q^* .

$$\begin{aligned} & 1 - \Phi\left(\sqrt{\beta}(q^* - \theta)\right) \\ &= \Phi\left(\sqrt{\beta}(\theta - q^*)\right) \end{aligned} \quad (3.9)$$

where $\Phi(\cdot)$ is the cumulative distribution function for the standard normal. Let θ^* be the common cost element at which this proportion is exactly z . Thus,

$$\Phi\left(\sqrt{\beta}(\theta^* - q^*)\right) = z \quad (3.10)$$

When $\theta \leq \theta^*$, the proportion of traders that sell is z or less. So, the question of whether $s \leq z$ boils down to the question of whether $\theta \leq \theta^*$. Conditional on $q_i = q^*$, the density over θ is normal with mean

$$\frac{\alpha\bar{\theta} + \beta q^*}{\alpha + \beta}$$

and precision $\alpha + \beta$. Thus, the probability that $\theta \leq \theta^*$ is the area under the conditional density of θ to the left of θ^* , namely

$$\Phi\left(\sqrt{\alpha + \beta}\left(\theta^* - \frac{\alpha\bar{\theta} + \beta q^*}{\alpha + \beta}\right)\right) \quad (3.11)$$

This expression gives $F(z|q^*)$. Substituting out θ^* by using (3.10) and rearranging, we can re-write (3.11) to give:

$$F(z|q^*) = \Phi\left(\frac{\alpha}{\sqrt{\alpha + \beta}}(q^* - \bar{\theta}) + \sqrt{\frac{\alpha + \beta}{\beta}}\Phi^{-1}(z)\right) \quad (3.12)$$

Differentiation of this expression with respect to z will give us the subjective density over s . Let us denote the density function over s by

$$f(s|q^*)$$

In the limit where $\beta \rightarrow \infty$, the idiosyncratic components of loss limits around θ shrink to zero, so that everyone's loss limit is given by θ . The limiting case of (3.12) when β becomes large is given by:

$$F(z|q^*) \rightarrow \Phi(\Phi^{-1}(z)) = z$$

so that $F(z|q^*)$ is the identity function. Since the cumulative distribution function over s is the identity function, the density over s is uniform. That is

$$f(s|q^*) = 1, \quad \text{for all } s$$

Thus, the solution for the threshold point v^* is unchanged from (3.5). Also, in the limiting case, when $v > v^*$, the density over the proportion who sell is the degenerate spike at 0, while when $v < v^*$, the density is the degenerate spike at 1. Hence, the switching strategy around v^* constitutes a symmetric equilibrium strategy.

4. Discussion

Liquidity black holes are associated with a sharp V-shaped price path for prices. The price at date 1 is given by $v - cs$, while the expected value of the asset at date 1 is v . Thus, the expected return from date 1 to date 2 is given by $\frac{v}{v - cs}$. In the limiting case, where the loss limits are perfectly correlated across traders, s takes the value 1 below v^* , and takes the value 0 above v^* . Thus, when there is a liquidity black hole at date 1, the expected return is

$$\frac{v}{v - c}$$

which is strictly larger than the actuarially fair rate of 1 for risk-neutral traders. The larger is c , the greater is the likely bounce in price. The parameter c is given by $c = \gamma\sigma^2$, where γ is the coefficient of absolute risk aversion and σ^2 is

the variance of the fundamentals. Since c gives the slope of the residual demand curve facing the active traders, we can interpret c as representing the degree of illiquidity of the market. The larger is c , the smaller is the capacity to absorb the selling pressure from the active traders. Thus, when a liquidity black hole comes into existence, a large c is associated with a sharper decline in prices, and a commensurate bounce back in prices in the final period.

Another implication of our model is that the trading volume at the time of the liquidity black hole and its aftermath will be considerable. When the market strikes the liquidity black hole, the whole of the asset holding in the risky asset changes hands from the risk-neutral short horizon traders to the risk-averse market making sector. Although we have not modelled the dynamics, we could envisage that immediately afterwards, once the loss limits have been adjusted down given the new price, there will be an immediate reversal of the trades in which the risky asset ends up back in the hands of the risk neutral traders once more. The large trading volume that is generated by these reversals will be associated with the sharp V-shaped price dynamics already noted. The association between the V-shaped pattern in prices and the large trading volume is consistent with the evidence found in Campbell, Grossman and Wang (1993) and Pastor and Stambaugh (2002).

Traders who are aware of their environment take account of limited liquidity in the market. The equilibrium strategies of the traders therefore also take account of the degree of illiquidity of the market. The solution for the threshold point (3.5) shows that when c increases, the gap between v^* and q_i increases also, as shown in figure 3.1. In other words, when c is large, a trader's trigger point v^* is much higher than his true loss limit q_i . The trader bails out at a much higher price than his loss limit because he is apprehensive about the effect of other traders bailing out. Just as in the run outcome in a bank run game, the traders in the

illiquid market bail out more aggressively when they fear the bailing out of other traders. Since in our model the efficient outcome is for the risk-neutral traders to hold the risky asset, the increase in c results in a greater welfare loss, ex ante.

This last point raises some thorny questions for regulatory policy. While the trigger-happy behaviour of the individual traders is optimal from the point of view of that trader alone, the resulting equilibrium is socially inefficient. In particular when the loss limit of one trader is raised, this has repercussions beyond that individual. For other traders in the market, the raising of the loss limit by one trader imposes an unwelcome negative externality in the form of a more volatile interim price. The natural response of the other traders would be to raise their own trading limits to match. The analogy here is with an arms race.

More generally, when the endogenous nature of price fluctuations is taken into account, the regulatory response to market risk may take on quite a different flavour from the orthodox approach using value at risk using historical prices. Danielsson, Shin and Zigrand (2002) and Danielsson and Shin (2002) explore these issues further.

APPENDIX.

In this appendix, we complete the argument for theorem 1 by showing that if $q_i > q^*$, trader i prefers to sell, and if $q_i < q^*$, trader i prefers to hold. For this step of the argument, we again appeal to the conditional density over s . Let us consider a variant of question (Q) for a trader whose loss limit exceeds the threshold point q^* . Thus, consider the following question.

“My loss limit is q_i and all others use the threshold strategy around q^* . What is the probability that s is less than z ?”

The answer to this question will yield $F(z|q_i, q^*)$ - the probability that the proportion of traders who sell is at most z , conditional on q_i when all others use the threshold strategy around q^* . When all traders are using the threshold strategy around q^* , the proportion of traders who sell is given by the proportion of traders whose loss limits have realizations to the right of q^* . When the common element of loss limits is θ , the individual loss limits are distributed uniformly over the interval $[\theta - \varepsilon, \theta + \varepsilon]$. The traders who sell are those whose loss limits are above q^* . Hence,

$$s = \frac{\theta + \varepsilon - q^*}{2\varepsilon}$$

When do we have $s < z$? This happens when θ is low enough, so that the area under the density to the right of q^* is squeezed to a size below z . There is a value of θ at which s is precisely equal to z . This is when $\theta = \theta^*$, where

$$\theta^* = q^* - \varepsilon + 2\varepsilon z$$

We have $s < z$ if and only if $\theta < \theta^*$. Thus, we can answer the question posed above by finding the posterior probability that $\theta < \theta^*$.

For this, we must turn to trader i 's posterior density over θ conditional on his loss limit being q_i . This posterior density is uniform over the interval $[q_i - \varepsilon, q_i + \varepsilon]$, since the ex ante distribution over θ is uniform and the idiosyncratic element of the loss limit is uniformly distributed around θ . The conditional probability that $\theta < \theta^*$ is then the area under the posterior density over θ to the left of θ^* . This is,

$$\begin{aligned} & \frac{\theta^* - (q_i - \varepsilon)}{2\varepsilon} & (4.1) \\ = & \frac{q^* - \varepsilon + 2\varepsilon z - (q_i - \varepsilon)}{2\varepsilon} \\ = & z + \frac{q^* - q_i}{2\varepsilon} \end{aligned}$$

This gives the cumulative distribution function $F(z|q_i, q^*)$, which falls under three cases.

$$F(z|q_i, q^*) = \begin{cases} 0 & \text{if } z + \frac{q^* - q_i}{2\varepsilon} < 0 \\ 1 & \text{if } z + \frac{q^* - q_i}{2\varepsilon} > 1 \\ z + \frac{q^* - q_i}{2\varepsilon} & \text{otherwise} \end{cases}$$

Hence, the corresponding density over s will, in general, have an atom at either $s = 0$ or $s = 1$. Thus, let us consider q_i where $q_i > q^*$. We show that trader i does strictly better by selling, than by holding. The conditional density over the half-open interval $s \in [0, 1)$ is given by

$$f(s|q_i, q^*) = \begin{cases} 0 & \text{if } s < \frac{q_i - q^*}{2\varepsilon} \\ 1 & \text{if } s \geq \frac{q_i - q^*}{2\varepsilon} \end{cases}$$

and there is an atom at $s = 1$ with weight $\frac{q_i - q^*}{2\varepsilon}$.

Meanwhile, from (2.7) and (2.8), the expected payoff advantage of holding relative to selling is given by

$$u - w = \begin{cases} \frac{1}{2}cs & \text{if } s \leq \hat{s}_i \\ -\frac{\hat{s}_i}{s} \left(v - \frac{1}{2}cs \right) & \text{if } s > \hat{s}_i \end{cases}$$

This payoff function satisfies the single-crossing property in that, $u - w$ is non-negative when $s \leq \hat{s}_i$, and is negative when $s > \hat{s}_i$. The density $f(s|q_i, q^*)$ can be obtained from the uniform density by transferring weight from the interval $[0, \frac{q_i - q^*}{2\varepsilon}]$ to the atom on point $s = 1$. Since $\int_0^1 (u(s) - w(s)) ds = 0$, we must have

$$\int_0^1 (u(s) - w(s)) f(s|q_i, q^*) ds < 0$$

Thus, the trader with loss limit $q_i > q^*$ strictly prefers to sell. There is an exactly analogous argument to show that the trader with loss limit $q_i < q^*$ strictly prefers to hold. This completes the argument for theorem 1.

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